ANTWOORDEN

Opgave 1

$\frac{2}{3}+\frac{7}{12}=\frac{15}{12}=\frac{5}{4}$ $\qquad \frac{2}{7}+\frac{3}{5}=\frac{31}{35}$ $\qquad -\frac{14}{60}+\frac{8}{21}-\frac{2}{30}=-\frac{49}{210}+\frac{80}{210}-\frac{14}{210}=\frac{17}{210}$

$\frac{\frac{3}{8}-\frac{1}{5}}{\frac{2}{3}}=\frac{21}{80}$ $\qquad \frac{\frac{6}{8}-\frac{2}{5}}{\frac{6}{9}}=\frac{\frac{3}{4}-\frac{2}{5}}{\frac{2}{3}}=\frac{\frac{15}{20}-\frac{8}{20}}{\frac{2}{3}}=\frac{\frac{7}{20}}{\frac{2}{3}}=\frac{7}{20}\cdot\frac{3}{2}=\frac{21}{40}$

$\frac{\frac{2}{4}-\frac{8}{5}}{-\frac{6}{7}}\cdot\frac{6}{11}=\frac{7}{10}$

Opgave 2

$8x^2\cdot y^0\cdot x^7=8x^9$ $\qquad \sqrt[3]{8}\cdot\sqrt[6]{x^{12}}=2x^2$ $\qquad 2\sqrt{a}\cdot 3\sqrt{b}\cdot a^{\frac{3}{2}}\cdot b^{\frac{5}{2}}=6a^2b^3$

$\sqrt[5]{x^2\cdot y}\cdot\sqrt[10]{x^{26}\cdot y^{18}}=x^3y^2$ $\qquad 5\cdot\sqrt[7]{a}\cdot\left(\sqrt{a+b^2}\right)^0\cdot\left(\sqrt[7]{a^3}\right)^2=5a$

$7x^2\sqrt{5}\cdot\sqrt{x^5}\cdot\sqrt{125}=7x^2\sqrt{5}\cdot\sqrt{x^5}\cdot\sqrt{25\cdot 5}=7x^2\sqrt{5}\cdot x^{\frac{5}{2}}\cdot 5\sqrt{5}==7\cdot x^{\frac{9}{2}}\cdot 25=175x^{4\frac{1}{2}}$

$\frac{16\sqrt{2}\cdot\sqrt{x^6}\cdot\sqrt{8}}{8\cdot\sqrt{x^{\frac{1}{2}}}\cdot\sqrt[4]{x^3}}=\frac{2^4\cdot 2^{\frac{1}{2}}\cdot x^{\frac{6}{2}}\cdot 2^{\frac{3}{2}}}{2^3\cdot x^{\frac{1}{4}}\cdot x^{\frac{3}{4}}}=\frac{2^4\cdot 2^2\cdot x^3}{2^3\cdot x}=2^3x^2=8x^2$

Opgave 3

$x^{-6} \cdot x^2 \cdot 3^2 = x^{2-6} \cdot 9 = \ x^{-4} \cdot 9 = \frac{9}{x^4}$

$\frac{y^3 \cdot x^{-7}}{y^{-2}} = y^{3-(-2)} \cdot x^{-7} = \frac{y^5}{x^7}$

$\frac{x^{2\frac{1}{2}} \cdot x^{-2}}{x^{1\frac{1}{2}} \cdot 2^5} = \frac{1}{2^5 x} = \frac{1}{32x}$

$\frac{x^{-p} \cdot y^2}{y^7} = x^{-p} \cdot y^{2-7} = x^{-p} \cdot y^{2-7} = \frac{1}{x^p y^5}$

$\frac{x^{2p} \cdot 5^{-3p}}{x^{-p-2}} = \frac{x^{2p-(-p-2)}}{5^{3p}} = \ \frac{x^{3p+2}}{5^{3p}}$

Opgave 4

$(x-y)^2 + \ 2\left(x^2 - \frac{1}{2}y^2\right) = \ x^2 - 2xy + y^2 + 2x^2 - y^2 \ = \ 3x^2 - 2xy$

$3\left(c^2 - d\right) + \ (2c-1)(3d+1) = \ 3c^2 - 3d + 6cd + 2c - 3d - 1 \ = \ 3c^2 - 6d + 6cd + 2c - 1$

$\left(a^2 - 5b\right)^2 - \frac{1}{2}\left(6a^4 - 2a^2b\right) = \ a^4 - 10a^2b + 25b^2 - 3a^4 + a^2b \ = \ -2a^4 - 9a^2b + 25b^2$

$(2x-2y)(1-x)(2x+3y)(1+x) \ = \ \left((2x)^2 + (-2y+3y)x - (-2y)\cdot(3y)\right)\cdot \left(1-x^2\right) =$

$= \ \left((2x)^2 + 2xy - 6y^2\right)\left(1-x^2\right) \ = \ 4x^2 + 2xy - 6y^2 - 4x^4 - x^3y + 6x^2y^2$

Opgave 5

$x^2 - y^2 = \ (x+y)(x-y)$

$x^2 + 5x + 6 = \ (x+2)(x+3)$

$a^2 - 6a + 9 \ = \ (a-3)^2$

$1 - a^8 = \ \left(1+a^4\right)\left(1+a^2\right)(1+a)(1-a)$

$4x^2 + 9y^2 + 12xy = (2x+3y)^2$

$25k^2 + 25mk + 6m^2 \quad = \quad (5k)^2 + (2m+3m)\cdot(5k) + 2m \cdot 3m \quad = \quad (5k+3m)(5k+2m)$

Opgave 6

Een handige formule is: $\frac{1}{\sqrt{A}} = \frac{1}{A}\sqrt{A}$

(want $\frac{1}{\sqrt{A}} = \frac{1}{\sqrt{A}} \cdot 1 = \frac{1}{\sqrt{A}} \cdot \frac{\sqrt{A}}{\sqrt{A}} = \frac{\sqrt{A}}{A} = \frac{1}{A}\sqrt{A}$ bijvoorbeeld: $\frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$)

$$2\sqrt{5} - \frac{3}{\sqrt{5}} = 2\sqrt{5} - \left(\frac{3}{\sqrt{5}}\right)\cdot\left(\frac{\sqrt{5}}{\sqrt{5}}\right) = 2\sqrt{5} - \frac{3}{5}\sqrt{5} = \left(2 - \frac{3}{5}\right)\sqrt{5} = \frac{7}{5}\sqrt{5}$$

$$\frac{\frac{1}{2}\sqrt{7}}{\frac{5}{4}\sqrt{8}} = \left(\tfrac{1}{2}\right)\cdot\left(\tfrac{4}{5}\right)\cdot\left(\sqrt{7}\right)\cdot\left(\tfrac{1}{8}\cdot\sqrt{8}\right) = \left(\tfrac{1}{2}\right)\cdot\left(\tfrac{4}{5}\right)\cdot\left(\tfrac{1}{8}\right)\cdot\left(\sqrt{7}\right)\cdot\left(\sqrt{4\cdot 2}\right) = \left(\tfrac{1}{20}\right)\cdot\left(\sqrt{7}\right)\cdot\left(2\sqrt{2}\right) = \tfrac{1}{10}\sqrt{14}$$

$$3\sqrt{20} - \tfrac{1}{2}\sqrt{5} = 3\cdot\sqrt{4}\cdot\sqrt{5} - \tfrac{1}{2}\sqrt{5} = 6\sqrt{5} - \tfrac{1}{2}\sqrt{5} = \tfrac{11}{2}\sqrt{5}$$

$$\frac{\sqrt{40}}{6\sqrt{81}} = \frac{\sqrt{4}\cdot\sqrt{10}}{6\cdot 9} = \left(\frac{2}{6\cdot 9}\right)\cdot\sqrt{10} = \tfrac{1}{27}\sqrt{10}$$

$$\frac{8\sqrt{125}}{3\sqrt{28}} = \frac{8\sqrt{25\cdot 5}}{3\sqrt{4\cdot 7}} = \tfrac{20}{21}\sqrt{35}$$

$$\frac{\sqrt{27} - 8\sqrt{3}}{\sqrt{81}} + \frac{7}{3\sqrt{3}} = \frac{3\sqrt{3} - 8\sqrt{3}}{9} + \frac{7}{3\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{-5\sqrt{3}}{9} + \frac{7\sqrt{3}}{9} = \tfrac{2}{9}\sqrt{3}$$

Opgave 7

Het eerste groepje van twee punten $(1, 8)$ en $(-3, 0)$

Vergelijking van de rechte lijn door deze punten $y = 2x + 6$

en de twee punten $\left(-\frac{3}{2}, 5\right)$ en $\left(-\frac{1}{8}, \frac{7}{8}\right)$ met vergelijking $y = -3x + \frac{1}{2}$

Opgave 8

$$\begin{aligned} 7x - 3y &= 5 \\ 6x - 5y &= -3 \end{aligned}$$

$x = 2 \quad y = 3$

$$\begin{aligned} x + 2y + z &= 8 \\ 2x + 3y - 2z &= 10 \\ 3x - 5y + 7z &= 6 \end{aligned}$$

$x = 3 \quad y = 2 \quad z = 1$

$$\begin{aligned} \tfrac{1}{2}x + \tfrac{2}{5}y &= 5 \\ x + \tfrac{3}{5}y &= 8 \end{aligned}$$

$x = 2 \quad y = 10$

$$\begin{aligned} 3x + y - 5z &= 4 \\ 2x + 4y + z &= 17 \\ x + 4z &= 6 \end{aligned}$$

$x = 2 \quad y = 3 \quad z = 1$

$$\begin{aligned} 7x + y\sqrt{2} &= 8\sqrt{2} \\ x\sqrt{3} + 8y\sqrt{6} &= 9\sqrt{6} \end{aligned}$$

$x = \sqrt{2} \quad y = 1$

$$\begin{aligned} 15x + 6y + 3z &= 24 \\ 9x - 9y - 6z &= -15 \\ 6x + 5y + 3z &= 17 \end{aligned}$$

$x = 1 \quad y = -2 \quad z = 7$

Opgave 9

$3x \le \tfrac{1}{8}(2 + 14x) - 1 \quad \Rightarrow \quad x \le -\tfrac{3}{5}$

$7x \le \tfrac{1}{2}(2 + 16x) - 4 \quad \Rightarrow \quad x \ge 3$

$6 - 7x > 3(3 - 2x) \quad \Rightarrow \quad 6 - 7x > 9 - 6x \quad \Rightarrow \quad 6x - 7x > 9 - 6$

$-x > 3 \quad \Rightarrow \quad x < -3$!!

Opgave 10

$$\frac{\sqrt[7]{a^5}\left(a^3 - \sqrt{b}\right)^0 a^{-6}}{\sqrt[7]{a^3}\,\sqrt[3]{b^6}} = \frac{\left(a^5\right)^{\frac{1}{7}} \cdot a^{-6}}{\left(a^3\right)^{\frac{1}{7}} \cdot \left(b^6\right)^{\frac{1}{3}}} = \frac{a^{\frac{5}{7}} \cdot a^{-6}}{a^{\frac{3}{7}} \cdot b^{\frac{6}{3}}} = \frac{a^{\frac{5}{7} - \frac{3}{7}} \cdot a^{-6}}{b^2} = \frac{a^{-\frac{40}{7}}}{b^2}$$

$p = -\tfrac{40}{7} \quad , \quad q = 2$

Opgave 11

$$\frac{1}{2a}+\frac{b}{a^2} \quad = \quad \frac{a+2b}{2a^2}$$

$$\frac{2\sqrt{y}}{\sqrt{x}}+\frac{5\sqrt{x}}{\sqrt{y}} \quad = \quad \frac{2\sqrt{y}}{\sqrt{x}}\cdot\left(\frac{\sqrt{y}}{\sqrt{y}}\right)+\frac{5\sqrt{x}}{\sqrt{y}}\cdot\left(\frac{\sqrt{x}}{\sqrt{x}}\right) \quad = \quad \frac{2y}{\sqrt{xy}}+\frac{5x}{\sqrt{xy}} \quad = \quad \frac{5x+2y}{\sqrt{xy}}$$

$$\frac{\frac{2}{x}-\frac{y}{x}}{\frac{y}{x^2}}\cdot\frac{y}{5x} \quad = \quad \frac{\frac{2-y}{x}}{\frac{y}{x^2}}\cdot\frac{y}{5x} \quad = \quad \left(\frac{2-y}{x}\right)\cdot\left(\frac{x^2}{y}\right)\cdot\left(\frac{y}{5x}\right) \quad = \quad \frac{2-y}{5}$$

Opgave 12

$$2x^2+3x=-7x-12 \quad \Rightarrow \quad 2x^2+10x+12=0 \quad \Rightarrow \quad x^2+5x+6=0$$

$$(x+2)(x+3)=0 \quad \Rightarrow \quad x+2=0 \ \vee \ x+3=0 \quad \Rightarrow \quad x=-2 \ \vee \ x=-3$$

$$\tfrac{1}{2}x+5=\frac{5x-3}{2} \quad \Rightarrow \quad x+10=5x-3 \quad \Rightarrow \quad x-5x=-10-3 \quad \Rightarrow \quad x= \ 3\tfrac{1}{4}$$

$$2x=7-x^2 \quad \Rightarrow \quad x^2+2x-7=0 \quad \Rightarrow \quad x_{1,2}=\frac{-2\pm\sqrt{4+28}}{2}$$

$$x_{1,2}=\frac{-2\pm\sqrt{32}}{2} \quad \Rightarrow \quad x_{1,2}=-1\pm\tfrac{1}{2}\sqrt{16\cdot 2} \quad \Rightarrow \quad x_{1,2}=-1\pm 2\sqrt{2}$$

$$x_1=-1+2\sqrt{2} \quad , \quad x_2=-1-2\sqrt{2}$$